# AP Statistics
# Test: t-Distribution for Means
# Instructor's Version

**Free Response: 2 questions, 52 points, 30 minutes**

**1. Some people believe that math ability and musical ability have a relationship — that is, people with musical ability also seem to do well in math. A group of researchers decides to test this theory. The researchers randomly select 100 musically gifted individuals and also 150 individuals with no musical talent. Each individual is given a test for math ability. The results are shown below:**

| Musically Gifted | Not Musically Gifted |
|---|---|
| $\bar{x} = 90.6$ | $\bar{x} = 88.4$ |
| $s = 1.8$ | $s = 2.6$ |
| $n = 100$ | $n = 150$ |

**A. Run an appropriate test to see if the mean math-ability score for those who are musically gifted is significantly higher than that of those who are not musically gifted. Use a 1% level of significance.** (22 points)

**Let $\mu_u$ = mean math-ability score for those who are musically gifted**
**Let $\mu_n$ = mean math-ability score for those who are not musically gifted**

**Answer:**
State hypotheses: $H_o : \mu_u = \mu_n$ OR $H_o : \mu_u - \mu_n = 0$

$H_\alpha : \mu_u > \mu_n$ $\quad$ $H_\alpha : \mu_u - \mu_n > 0$

Name of test: *t*-test for a difference of two means

Conditions:
1. Both samples are random: stated in the problem.
2. Both samples are large: $n_u = 100$ and $n_n = 150$ $(n > 30)$.
3. Samples are independent: It is reasonable to assume that the two samples are chosen independently of each other.

Mechanics: $t = 7.9023$
*p*-value = 4.4925 X 10^-14
df = 247.6253 (from calculator) or *n* - 1 = 99 (using the smaller sample)
tcdf(7.9023,E99,99) = 1.9419X10^-12
tcdf(7.9023, E99,247.6253) = 4.4925 X 10^-14

Decision: Reject $H_o$ since *p*-value, $4.4925 \cdot 10^{-14}$ is less than $\alpha = 0.01$

Conclusion: The sample provides strong evidence to support that the mean math-ability score for those who are musically gifted is greater than that of those who are not musically gifted.

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**Scoring notes:**
Award 2 points for a correct pair of hypotheses.
Award 2 points for the correct name of the test. If the student fails to name the test, but shows the calculation for *t* in the mechanics section, then he or she receives these points.
Award 2 points for each correct condition (for a maximum of 6 points).
Award 2 points each for a correct *t* test statistic, *p*-value, and df (for a maximum of 6 points).

$$t = \frac{90.6 - 88.4}{\sqrt{0.0324 + 0.0451}} = \frac{2.2}{0.2784} 7.9023 \text{ (by hand)}$$

Award 2 points for correct decision with reason. Award 1 point if no reason is given.
Award 4 points for a correct conclusion with context. Student should not state that those who are musically gifted are better math students.

**B. Find and interpret a 90% confidence interval for $\mu_u - \mu_n$. Provide a conclusion pertaining to hypothesis.** (10 points)

**Answer:**
90% confidence interval is (1.7405, 2.6595) (calculator solution)

Without calculator t* = 1.660 for degrees of freedom = 100.
2.2+-1.660(.2784) = (1.7379, 2.6621)

Note: Calculator solution uses more accurate degrees of freedom resulting in smaller interval

Interpretation: It can be estimated with 90% confidence that the true mean population difference in math ability score for those who are musically gifted and those who are not is between 1.7405 and 2.6595.

Conclusion: There is enough evidence to reject the null hypothesis because 0 is not in the interval.

**Scoring notes:**
Award 2 points for a correct confidence interval, 4 points for a correct interpretation with context, and 4 points for conclusion. No credit if no context is given.

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**2. It is believed that the mean number of dogs owned by dog-owning households is 2.1 dogs. An SRS of 35 dog owners has $\bar{x} = 2$ and $s = .29$. Carry out an appropriate test to see if the mean number of dogs owned by dog-owning households is something other than 2.1 dogs. Use a 1% level of significance.** (20 points)

**Answer:**

Define parameter of interest: $\mu$, which is the mean number of dogs owned by the population of all dog owners

Hypotheses: $H_o : \mu = 2.1$
$H_\alpha : \mu \neq 2.1$

Name of test: *t*-test for one mean

Conditions:
1. SRS is stated in problem
2. Large *n*, *n* = 35 (n > 30)
3. Observations are independent: We must be willing to treat the 35 dog owners as independent observations from the population of dog owners. This is a reasonable assumption.

Mechanics: *t* = -2.0400
*p*-value = 0.0492
df = n – 1 = 34

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = (2\text{-}2.1)/(1.29/5.916) = -2.04$$

1-tcdf(-2.04,2.04,34) = 0.0492

Decision: Fail to reject $H_o$ because *p*-value, 0.0492 is > $\alpha = 0.01$

Conclusion: The sample does not provide sufficient evidence that the mean number of dogs owned by dog-owners is something other than 2.1.

**Scoring notes:**
Award 1 point for correctly identifying the parameter of interest.
Award 2 points for a correct pair of hypotheses.
Award 1 point for correct test name. If the student fails to name the test, but shows the calculation for *t* in the mechanics section, then he or she receives these points.
Award 2 points for each correct condition (maximum of 6 points).
Award 2 points for each correct mechanics (maximum of 6 points).
Award 2 points for correct decision with reason.
Award 2 points for correct conclusion with context.

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